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ANALYSIS OF A DISTRIBUTED SUPERCONDUCTIVE ENERGY CONVERTER

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ANALYSIS OF A DISTRIBUTED SUPERCONDUCTIVE ENERGY CONVERTER

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SUMMARY

An equivalent circuit for a distributed superconductive energy converter, the operation of which is based on the principle of flux conservation in superconducting loops, is presented. The Laplace transformation is used to analyze the circuit, and the solutions are examined in order to optimize the operation of the device. While previously only dc operation of such a device has been realized, it is shown herein that under certain conditions a low frequency ac mode can be achieved.

author

INTRODUCTION

The principle of flux conservation in a multiply connected superconductor has been utilized as the basis of a mechanoelectrical energy converter.[1] In the cited reference the flux of a magnet is trapped in a normal hole of a triply connected superconductor (Fig. 1). If the magnet is rotated mechanically, flux from the magnet continuously links the load inductor since the flux is conserved in each loop of the triply connected system. Current is thus built up in the load inductor. The limit of the device is the critical current of the load inductor since above this current the load becomes normally conducting.

Since the change of load current per cycle of magnet rotation α is small, the converter is essentially a dc generator. In order to realize an increase of α and to optimize the operation of the converter, an analysis of this type of device is needed.

The theoretical analysis of the converter shown in Fig. 1 is exceedingly difficult because of the complexity of the current distribution in the superconducting disk and the lack of symmetry of the device as a whole. The lack of symmetry may be overcome by considering a distributed superconductive energy converter (DSEC) shown in Fig. 2 (for simplicity only threefold symmetry is shown). The angle between two adjacent inductors (and two adjacent holes) is 120°.

The purpose of this paper is to present an "equivalent circuit" analysis for a DSEC with N-fold symmetry. The requirements for optimum operation of the DSEC will be derived. Very low frequency ac operation will be shown to be possible, and a typical design for such a mode will be discussed.[2]

ANALYSIS FOR THE DSEC

In Figs. 3(a) and (b), one section of an N-fold symmetric DSEC is shown. In the N-fold symmetric DSEC, the N load inductors are equally spaced (the angle between two adjacent inductors is 360°/N). Similarly the normally conducting holes remain equally spaced during the magnet rotation. The smallest spacing between the perimeters of two adjacent holes will be assumed to be smaller than the hole radius. In this case, the current will be confined to narrow paths, and an equivalent circuit analysis is justified.

An equivalent circuit for one section of the N-fold symmetric DSEC is shown in Fig. 3(c). The inductances associated with the current paths around the normally conducting holes are L_S and L_P. As a hole passes under L_0, L_1 varies from zero to L_S, and L_2 varies from L_S to zero. Assume that $n - 1$ holes have passed under L_0. The dynamics of the passage of the nth hole will now be described. The current in each element of the equivalent circuit (and the values of L_1 and L_2) before the hole passes under L_0 will be denoted by the additional subscript B. After the nth hole has passed under L_0, the subscript A will be used. When neither A nor B is used it will be understood that the quantity is expressed at an arbitrary time.

Because of the periodicity of the structure in question, it is evident from Fig. 3(c) that at every instant of time

$$i_1 = i_4 \tag{1}$$

and

$$v_2 = v_6 \tag{1'}$$

[1] J. Volger and P. S. Admiraal, "A Dynamo for Generating a Persistent Current in a Superconducting Circuit," Physics Letters, vol. 2, p. 257, 1962.

[2] During the writing of this paper, a similar symmetrical design was presented by S. L. Wipf, "A Superconducting D.C. Generator," Scientific Paper 63-128-280-PS Westinghouse Research Laboratories. Since only a rough analysis was used to explain the experimental results, the analysis in this paper can be utilized to make the Wipf design more efficient and adapt this type of generator for ac operation.

If Kirchoff's current law is used, it is obvious that

$$i_1 = i_2 + i_3 \tag{2}$$

and

$$i_3 = i_4 + i_0 \tag{3}$$

If equation (1') and Kirchoff's voltage law are used, it can be shown that

$$L_0 \frac{di_0}{dt} - \frac{d}{dt}(L_2 i_4) - L_P \frac{di_2}{dt} = 0 \tag{4'}$$

Similarly,

$$L_P \frac{di_2}{dt} - L_0 \frac{di_0}{dt} - \frac{d}{dt}(L_1 i_3) = 0 \tag{5'}$$

There are two inductors, each of value L_0, adjacent to the L_0 under consideration in Fig. 3(c). Considering only these two inductors, denote the one under which the nth hole passes first as L_{01} and the other as L_{02}. Define t_B as the time when the nth hole is midway between L_{01} and L_0 and t_A as the time when the nth hole is midway between L_0 and L_{02}. Integration with respect to time, of equations (4') and (5') from t_B to t_A yields, respectively (as mentioned above $L_{1A} = L_{2B} = L_S$ and $L_{1B} = L_{2A} = 0$),

$$L_0(i_{0A} - i_{0B}) + L_S i_{4B} - L_P(i_{2A} - i_{2B}) = 0 \tag{4}$$

$$L_P(i_{2A} - i_{2B}) - L_0(i_{0A} - i_{0B}) - L_S i_{3A} = 0 \tag{5}$$

Simultaneous solution of equations (1) - (5) yields the following results

$$i_{0A} = i_{0B} - \eta i_{4B} \tag{6}$$

$$i_{1A} = -i_{0B} + (1 + \eta) i_{4B} \tag{7}$$

$$i_{2A} = -i_{0B} + \eta i_{4B} \tag{8}$$

$$i_{3A} = i_{4B} \tag{9}$$

$$i_{4A} = -i_{0B} + (1 + \eta) i_{4B} \tag{11}$$

where $\eta = L_S/(L_0 + L_P)$

As is apparent from equations (4) and (5), the results are explicitly independent of time.

It is now convenient to replace the subscript A by the argument n and B by the argument n - 1. For example, equation (6) becomes

$$i_0(n) = i_0(n - 1) - \eta i_4(n - 1) \tag{6'}$$

From equations (6) and (10), the difference equations for $i_0(n)$ and $i_4(n)$ are

$$i_0(n + 2) = (2 + \eta) i_0(n + 1) - i_0(n) \tag{11}$$

and

$$i_4(n + 2) = (2 + \eta) i_4(n + 1) - i_4(n) \tag{12}$$

In order to solve equations (11) and (12) for $i_0(n)$ and $i_4(n)$, respectively, it is necessary to introduce the initial conditions of the system. It will be assumed that before the holes begin to move, the load current is zero. The magnetic flux Φ through each hole is equivalent to a persistent current I_0 flowing around each hole. Thus

$$\begin{matrix} i_0(0) = 0 & i_1(0) = I_0 & i_2(0) = 0 \\ & i_3(0) = I_0 \qquad i_4(0) = I_0 & \end{matrix} \tag{13}$$

where

$$\Phi = L_S I_0 \tag{14}$$

It should be noted that $i_2(0)$ is zero since the persistent currents of two adjacent holes cancel in L_P. Applying the initial conditions given in equations (13) to equations (6) and (10) indicates that

$$\left.\begin{matrix} i_0(0) = 0 & i_0(1) = -\eta I_0 \\ i_4(0) = I_0 & i_4(1) = (1 + \eta) I_0 \end{matrix}\right\} \tag{15}$$

If the techniques of the Laplace transformation as applied to difference equations are used, it can be shown that[3]

$$i_0(n) = -\eta I_0 \left(\frac{x_1^n - x_0^n}{x_1 - x_0} \right) \tag{16}$$

and

$$i_4(n) = I_0 \left[\frac{\left(x_1^{n+1} - x_1^n\right) - \left(x_0^{n+1} - x_0^n\right)}{(x_1 - x_0)} \right] \tag{17}$$

where

$$x_{\substack{0\\1}} = \left(1 + \frac{\eta}{2}\right) \pm \sqrt{\left(1 + \frac{\eta}{2}\right)^2 - 1}$$

The validity of equations (16) and (17) can be checked by substituting into the applicable difference equations and noting that the initial conditions are satisfied. (The algebra is simplified if it is realized that $x_0 x_1 = 1$.)

[3] H. S. Carslaw and J. C. Jaeger, "Operational Methods in Applied Mathematics," Oxford University Press, 2nd Edition, 1947.

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Equations (1), (3), (16) and (17) yield the following expressions:

$$i_2(n) = \eta I_0\left(\frac{x_1^n - x_0^n}{x_1 - x_0}\right) \tag{18}$$

$$i_1(n) = I_0\left[\frac{\left(x_1^{n+1} - x_1^n\right) - \left(x_0^{n+1} - x_0^n\right)}{\left(x_1 - x_0\right)}\right] \tag{19}$$

and

$$i_3(n) = I_0\left\{\frac{\left[x_1^{n+1} - (1+\eta)x_1^n\right] - \left[x_0^{n+1} - (1+\eta)x_0^n\right]}{\left(x_1 - x_0\right)}\right\}$$

$$= I_0\left[\frac{\left(x_1^n - x_1^{n-1}\right) - \left(x_0^n - x_0^{n-1}\right)}{x_1 - x_0}\right] \tag{20}$$

In the appendix, analytic expressions for all the currents are derived for the case $i(k')$ where $n - 1 < k' \leq n$. For all the design criteria discussed in this paper, however, only equations (16) - (20) are necessary.

In order to illustrate the use of equations (16) - (20), two limiting cases of interest will be discussed, namely, $\eta \ll 1$ and $\eta \gg 1$.

Case One: $\eta \ll 1$

In this approximation,

$$x_{\substack{0\\1}} \approx 1 \pm \sqrt{\eta} \tag{21}$$

and

$$x_{\substack{0\\1}}^{\ n} \approx 1 \pm n\sqrt{\eta} \tag{22}$$

Thus,

$$i_0(n) = -\,n\eta I_0 \tag{23}$$

If $L_0 \gg L_P$, it is found, by combining equations (14) and (23), that

$$i_0(n) = -\frac{n\Phi}{L_0} \tag{24}$$

The result that $i_0(n)$ is proportional to n was observed experimentally by Volger and Admiraal.[1] If Φ_0 is the flux in L_0, then

$$\Phi_0(n) = n\Phi \tag{25}$$

which agrees quantitatively with their observations.

It is valid to use the results of the analysis of the DSEC for the case of the Volger-Admiraal dynamo since for $\eta \ll 1$ and $L_0 \gg L_P$ the flux built up in the load is independent of the current distribution in the disk (see eqs. (24) and (25)).

Equation (25) was found and experimentally verified by Wipf,[2] who used an analysis based on approximations that are equivalent to the assumptions that $\eta \ll 1$ and $L_0 \gg L_P$. The advantage of the analysis given herein is that it is possible to investigate the case in which $\eta \gg 1$, a situation not considered to date. It is, however, encouraging that the results of the present general analysis reduce to the correct lower limit.

Case Two: $\eta \gg 1$

In this approximation,

$$x_0 \approx \eta \tag{26}$$

$$x_1 \gtrsim 0 \tag{27}$$

Thus,

$$i_0(n) = -\eta^n I_0 \tag{28}$$

for $n \geq 1$.

Equation (28) can be transformed to a flux equation. With equation (14) it can be shown that since $\Phi_0 = L_0 i_0$,

$$\Phi_0(n) = \eta^n\ \Phi\left(\frac{L_0}{L_S}\right) = \eta^{n-1}\left(\frac{L_0}{L_P + L_0}\right)\Phi \tag{29}$$

for $n \geq 1$.

In the next section the relative merits of the designs in which $\eta \ll 1$ and $\eta \gg 1$ will be discussed.

DESIGN OF THE DSEC

It is convenient to define a converter efficiency E where

$$E = \frac{i_0(n+1)}{i_0(n)} \tag{30}$$

The ratio E is a measure of the change in load current per unit mechanical rotation of the magnets α. Larger values of E correspond to greater mechanoelectrical frequency response α. It is easily seen by combining equations (23) and (30) that for large n

$$E_{\eta \ll 1} \gtrsim 1 \tag{31}$$

while combining equations (28) and (30) yields the result that

$$E_{\eta \gg 1} = \eta \gg 1 \tag{32}$$

for all values of n. Consequently, the conversion of mechanical to electrical energy is more efficient for the case $\eta \gg 1$.

Since $E_{\eta \gg 1} \gg E_{\eta \ll 1}$ for all n, more current can be generated after a given number of turns of the magnet array. By oscillating the magnet array about an equilibrium position, an ac load current can be generated. The magnitude and frequency of the load current will be estimated in the DISCUSSION section.

To physically realize the $\eta \gg 1$ case it is necessary that

$$L_S \gg L_P + L_O \tag{34}$$

In the physical system shown in Fig. 2, however,

$$L_O \gg L_P . L_S \tag{35}$$

since more magnetic energy is stored in L_O.

Thus it seems quite difficult to satisfy equation (34). There are at least two possible design modifications that tend to alleviate this difficulty.

The first modification, shown in Fig. 4(a), makes use of a rectangular hole. This tends to increase L_S since L_S is a measure of the current flowing in a circumferential direction in the disk. By properly shaping the magnets, the rectangular geometry is easily realized. A magnetically permeable coating is also indicated. This coating tends to increase L_S and L_P more than it increases L_O. While the condition $\eta \gg 1$ will probably not be attained, a larger value than that of the design in Fig. 2 seems likely.

The second modification, shown in Fig. 4(b), consists of using inductive elements in place of the disk. All wires are high critical field superconductors except for the shaded portions, which simulate holes. The advantage of this scheme is that the values of L_S and L_P can be controlled. It is expected that this design will be superior to the first modification.

CONCLUSIONS

An analysis of the equivalent circuit of the DSEC indicates the possibility of efficient mechanoelectrical energy conversion. The equivalent circuit used, however, is only an approximaation to the physical converter. The approximation is valid if the current is confined to narrow paths, approximating lumped element behavior. The analysis does not take into account hysteresis effects; however, this effect is judged to be small and in any case does not influence the buildup of flux in the load but only decreases the amount of recoverable energy. It was also assumed that the response of the hole to the motion of the magnet is instantaneous. Such an approximation is valid for rotational speeds that are less than 50 rpm.[4] Wipf[2] has shown that for a design similar to that shown in Fig. 2, a current of 40 amperes can be generated after 3600 turns. From equations (31) and (32) it appears that this amount of current could be generated in a fraction of a turn if the second modified design proposed herein is used. Thus, if the magnet array oscillates about an equilibrium position as was mentioned earlier, it is expected that ac currents of the order of 40 amperes and a frequency of 1 cps can be realized.

APPENDIX - EVALUATION OF i(k')

In order to find $i(k')$ when $n-1 < k' < n$ (when the hole is in an intermediate position under L_O) it is merely necessary to note that

$$L_{1A} = kL_S \qquad L_{2A} = (1-k)L_S \tag{A1}$$

where now the subscript A refers to the instant that the hole is in an intermediate position between L_{O1} and L_{O2}. Equations (4) and (5) become, respectively

$$L_O(i_{OA} - i_{OB}) - (1-k)L_S i_{4A} + L_S i_{4B} - L_P(i_{2A} - i_{2B}) = 0 \tag{A2}$$

$$L_P(i_{2A} - i_{2B}) - L_O(i_{OA} - i_{OB}) - kL_S i_{3A} = 0 \tag{A3}$$

Solving as before shows that

$$i_O(k') = -i_2(k') = \frac{i_O(n-1) - k\eta i_4(n-1)}{1 + \eta k(1-k)} \tag{A4}$$

$$i_1(k') = i_4(k')$$

$$= \frac{(1 + k\eta)i_4(n-1) - ki_O(n-1)}{1 + \eta k(1-k)} \tag{A5}$$

$$i_3(k') = \frac{i_4(n-1) + (1-k)i_O(n-1)}{1 + \eta k(1-k)} \tag{A6}$$

where $k' = n - 1 + k$ and $0 < k \leq 1$; $i_O(n-1)$ and $i_4(n-1)$ are given by equations (17) and (18).

[4] J. Vogler and J. van Suchtelen, "Induction of Heavy Persistent Currents," Conference on High Magnetic Fields, Their Production and Their Applications, Clarendon Laboratory, University of Oxford, July 10-12, 1963.

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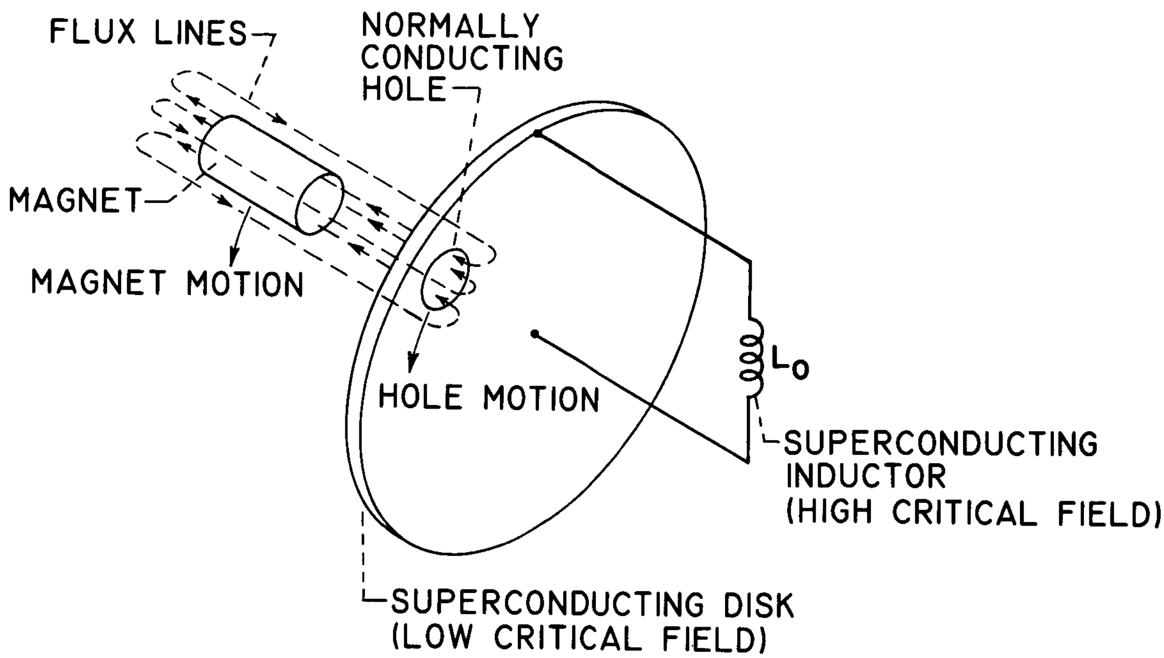


Figure 1. - Volger - Admiraal generator.

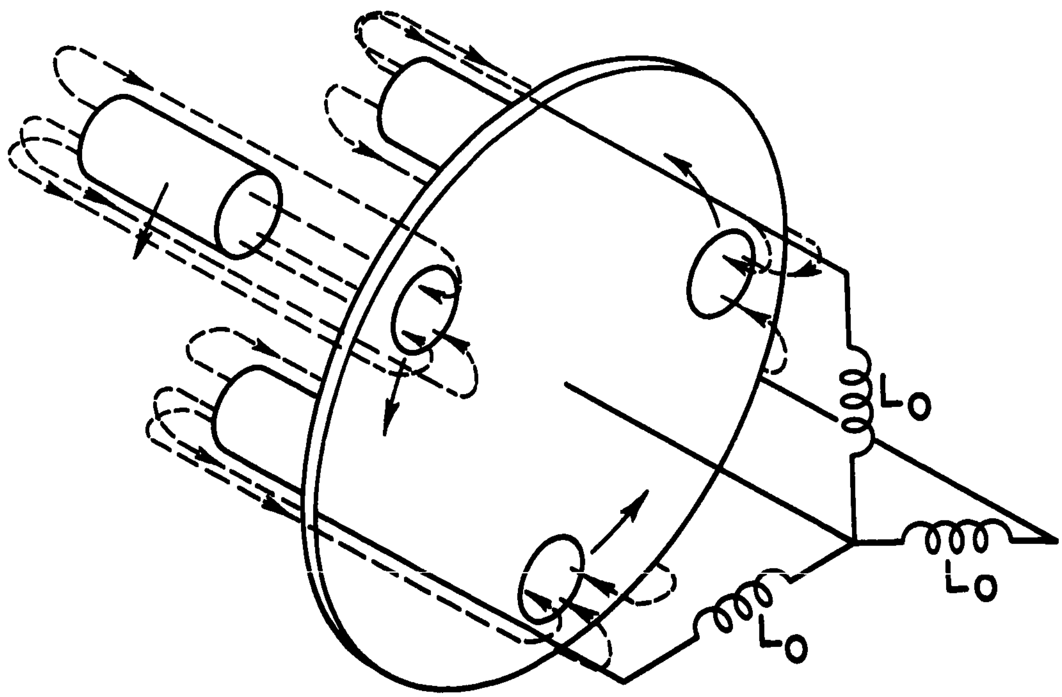
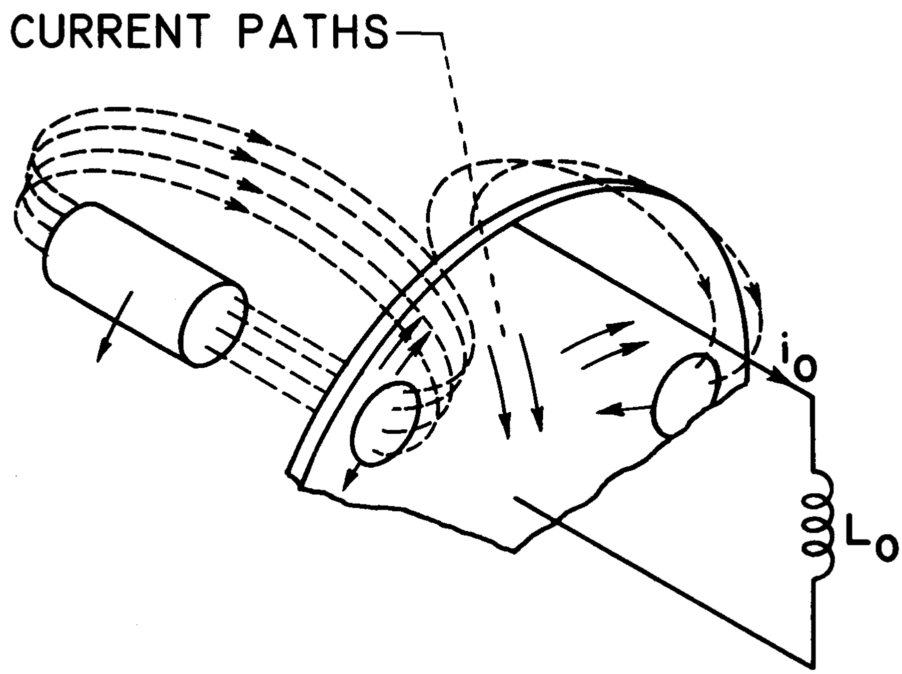
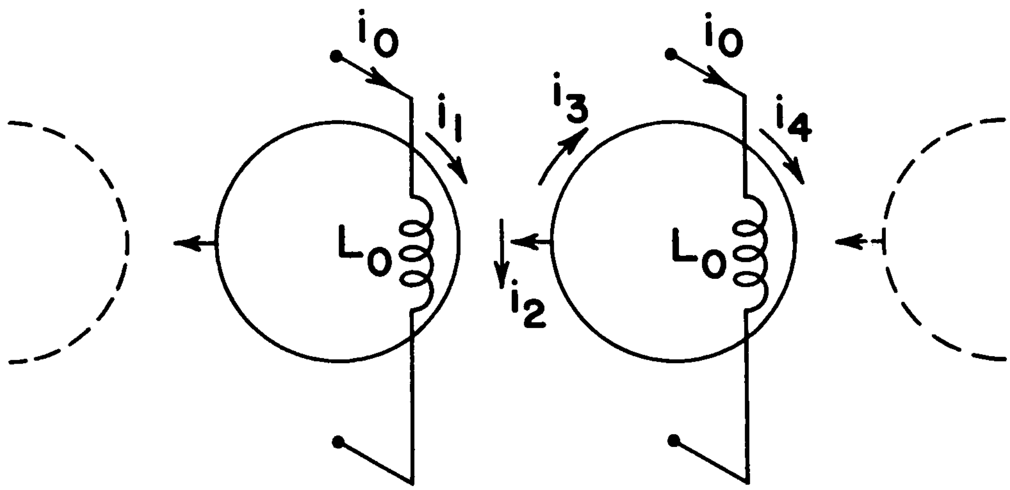


Figure 2. - DSEC: threefold symmetric.



(a) Pictorial view.

Figure 3. - Section of an N-fold symmetric DSEC.



(b) Schematic view.

Figure 3. - Continued. Section of an N-fold symmetric DSEC.

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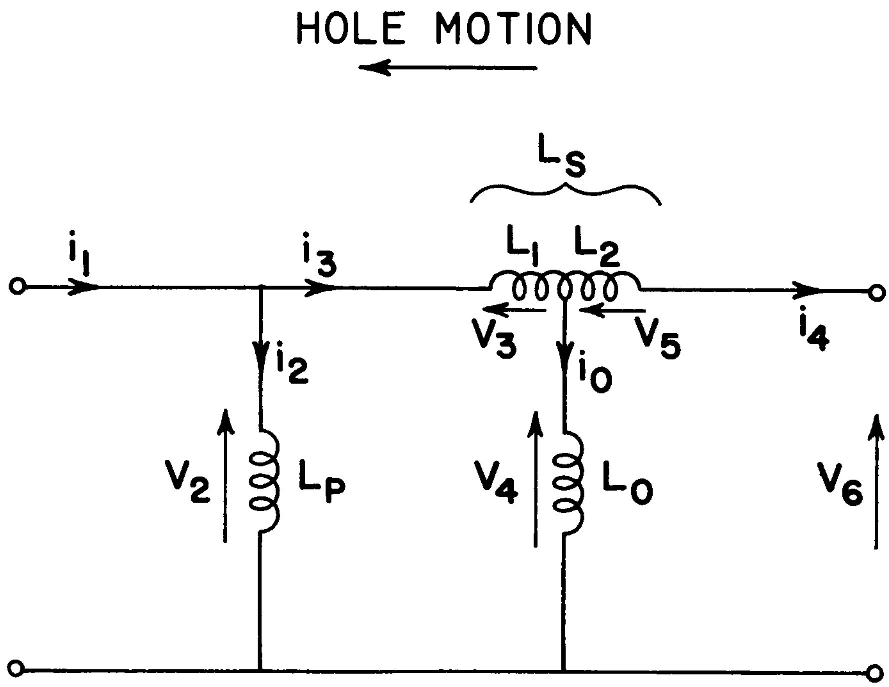
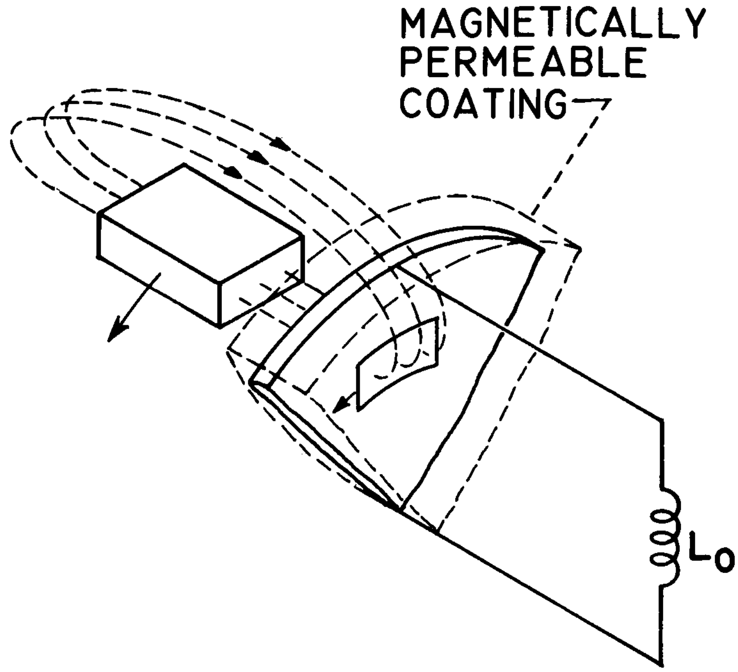


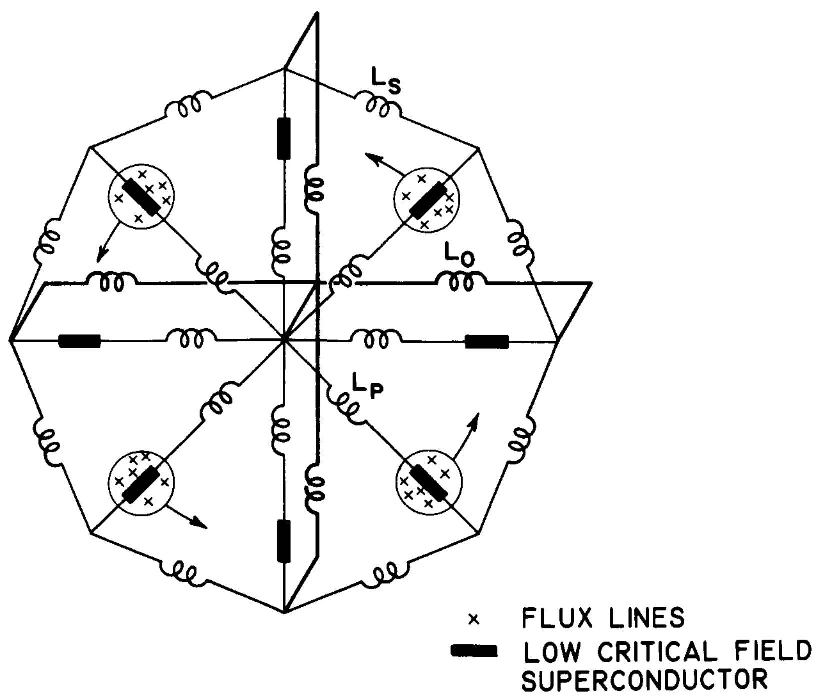
(c) Circuit equivalent.

Figure 3. - Concluded. Section of an N-fold symmetric DSEC.



(a) Modification one.

Figure 4. - Improved DSEC design.



(b) Modification two.

Figure 4. - Concluded. Improved DSEC design.

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